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On the nature of prime numbers

by Birke Heeren

This paper by a non mathematician scientist describes several starting points for further research in prime number theory. It describes new tools like Prime Faculty. The Periodic sieve algorithm (Psa) is developed in detail. Further impetus is given with a prime number function, Latisses, six pack structures (four pattern letters), expanding fractals, and calculating with infinite sets. Proofs about prime triplets and prime gaps are provided.

Keywords prime numbers, Erastothenes type sieves, Psa, algorithm, prime number functions, four pattern letters, six pack structures, prime number function theorem, Latisses, prime triplets, prime gaps, expanding fractals, Primzahlmaschine, infinite sets

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Chapter 1 Introduction

The theory of prime numbers has been developed for more than 2300 years. Euclid proved that there are an infinite number of prime numbers. (Simon Singh 2006).

Prime numbers are used in cryptography and are important to other fields of science and technology. Even though the special prime number “one” is often excluded from prime number theories it neatly fits into the nature of prime numbers as described in this paper.

This paper is almost devoid of references because the prerequisite knowledge is either trivial or developed from the very beginning. Also the author as an oceanographer, biologist and programmer confesses to lack the mathematical knowledge to understand the pure mathematical literature on prime numbers. Maybe this lack of standard knowledge opened the way to fresh and new insights into an age old problem “the nature” of prime numbers.

Chapter 2 Tools

The natural numbers $\mathbb{N}$ exclude zero in this paper. For the purpose of describing the nature of prime numbers, $\mathbb{N}$ is divided into three sets $\mathbb{A}$, $\mathbb{B}$, and $\mathbb{C}$ in such way that

$$\mathbb{A} + \mathbb{B} + \mathbb{C} := \mathbb{N} \tag{I}$$

Each $n \in \mathbb{N}$ is matched with one specific set $\mathbb{A}$, one specific set $\mathbb{B}$, and one specific set $\mathbb{C}$. The matching is denoted by the index n, thus the sets are written as $\mathbb{A}_n$, $\mathbb{B}_n$, and $\mathbb{C}_n$ with $n \in \mathbb{N}$. **II**

$\mathbb{B}_n$ is defined to contain exactly one element:

$$\mathbb{B}_n := \{\, x \mid x \in \mathbb{N} \text{ and } x = n \,\} \qquad \text{with } n \in \mathbb{N} \tag{III}$$

$\mathbb{A}_n$ is defined:

$$\mathbb{A}_n := \{\, x \mid x \in \mathbb{N} \text{ and } x < n \,\} \qquad \text{with } n \in \mathbb{N} \tag{IV}$$

$\mathbb{C}_n$ is defined:

$$\mathbb{C}_n := \{\, x \mid x \in \mathbb{N} \text{ and } x > n \,\} \qquad \text{with } n \in \mathbb{N} \tag{V}$$

The sets $\mathbb{A}_n$, $\mathbb{B}_n$, and $\mathbb{C}_n$ are used to move through all natural numbers $\mathbb{N}$ in a step wise fashion (see E1) with $\mathbb{B}_n$ containing the element n equal in value to the step number n. The element in $\mathbb{B}_n$ is thus also called "step number".

Examples

$\mathbb{A}_1 = \varnothing$	$\mathbb{B}_1 = \{ 1 \}$	$\mathbb{C}_1 = \{ 2, 3, \dots \infty \}$	
$\mathbb{A}_2 = \{ 1 \}$	$\mathbb{B}_2 = \{ 2 \}$	$\mathbb{C}_2 = \{ 3, 4, \dots \infty \}$	
$\mathbb{A}_3 = \{ 1, 2 \}$	$\mathbb{B}_3 = \{ 3 \}$	$\mathbb{C}_3 = \{ 4, 5, \dots \infty \}$	E1
$\mathbb{A}_4 = \{ 1, 2, 3 \}$	$\mathbb{B}_4 = \{ 4 \}$	$\mathbb{C}_4 = \{ 5, 6, \dots \infty \}$	
$\mathbb{A}_5 = \{ 1, 2, 3, 4 \}$	$\mathbb{B}_5 = \{ 5 \}$	$\mathbb{C}_5 = \{ 6, 7, \dots \infty \}$	

Stepping through $\mathbb{N}$ as described will be used for the Periodic sieve algorithm (Psa) described in chapter 3. For the purpose of this algorithm a starting state that precedes the first step in $\mathbb{N}$ is defined as follows.

$$\mathbb{A}_{start} := \varnothing \qquad \mathbb{B}_{start} := \varnothing \qquad \mathbb{C}_{start} := \{ 1, 2, \dots \infty \} = \mathbb{N} \tag{VI}$$

In the starting state all natural numbers $n \in \mathbb{N}$ are contained in set $\mathbb{C}_{start}$ and in accordance with equation I the sets $\mathbb{A}_{start}$ and $\mathbb{B}_{start}$ are empty sets.

For use in the Psa in chapter 3 each natural number $n \in \mathbb{N}$ from sets $\mathbb{A}_n$, $\mathbb{B}_n$, and $\mathbb{C}_n$ is assigned one of the following three types:

- Type P: Numbers determined to be prime are denoted by letter P. VII
- Type M: Numbers determined to be a multiple of a prime number are called "multiples" and are denoted by letter M. VIII
- Type L: Numbers not yet determined are called “Lücke” (= space) and are denoted by letter L. IX

In addition to sets $\mathbb{A}_n$, $\mathbb{B}_n$, and $\mathbb{C}_n$ containing natural numbers, further sets $\mathbb{AP}_n$, $\mathbb{BP}_n$, and $\mathbb{CP}_n$ are defined which contain the corressponding type information. These sets are called "pattern sets".

$\mathbb{AP}_n$ is defined:

Each set $\mathbb{AP}_n$ is matched with set $\mathbb{A}_n$ in such way that each type element in $\mathbb{AP}_n$ is matched with the corresponding natural number element in $\mathbb{A}_n$ with $n \in \mathbb{N}$. X

$\mathbb{BP}_n$ is defined:

Each set $\mathbb{BP}_n$ is matched with set $\mathbb{B}_n$ in such way that each type element in $\mathbb{BP}_n$ is matched with the corresponding natural number element in $\mathbb{B}_n$ with $n \in \mathbb{N}$. XI

$\mathbb{CP}_n$ is defined:

Each set $\mathbb{CP}_n$ is matched with set $\mathbb{C}_n$ in such way that each type element in $\mathbb{CP}_n$ is matched with the corresponding natural number element in $\mathbb{C}_n$ with $n \in \mathbb{N}$. XII

Pattern sets $\mathbb{AP}_{start}$, $\mathbb{BP}_{start}$, and $\mathbb{CP}_{start}$ are defined analogous. XIII

In order to match correctly each natural number with its type information, sets $\mathbb{A}_n$ and $\mathbb{C}_n$ are written in an ordered manner with the smallest value on the left. Thus each element has all elements of larger value to its right. Sets $\mathbb{AP}_n$ and $\mathbb{CP}_n$ then have a matching order by way of the Psa.

The examples E2 are results of the Psa in chapter 3. These examples illustrate some of the basic ideas of this paper.

$\mathbb{AP}_{start} := \varnothing$	$\mathbb{BP}_{start} := \varnothing$	$\mathbb{CP}_{start} := \{ L, L, \ldots \infty \} = \{ \overline{L} \}$	
$\mathbb{AP}_1 = \varnothing$	$\mathbb{BP}_1 = \{ P \}$	$\mathbb{CP}_1 = \{ \overline{L} \}$	
$\mathbb{AP}_2 = \{ P \}$	$\mathbb{BP}_2 = \{ P \}$	$\mathbb{CP}_2 = \{ \overline{LM} \}$	E2
$\mathbb{AP}_3 = \{ P, P \}$	$\mathbb{BP}_3 = \{ P \}$	$\mathbb{CP}_3 = \{ \overline{MLMLMM} \}$	
$\mathbb{AP}_4 = \{ P, P, P \}$	$\mathbb{BP}_4 = \{ M \}$	$\mathbb{CP}_4 = \{ \overline{LMLMMM} \}$	

The first basic idea is that patterns of $\mathbb{AP}_n$ and $\mathbb{CP}_n$ contain information about the natural numbers and thus also prime numbers in $\mathbb{A}_n$ and $\mathbb{C}_n$.

The second basic idea is to step through $\mathbb{N}$ from 1 to infinity and thereby also to cover all prime numbers from 1 to infinity.

The third basic idea is that each $\mathbb{CP}_n$ contains a periodic and thus infinite pattern that contains information about all possible prime numbers (Hüllkurve from Physics) greater than the step number (well defined lower bound). And therefore information on the nature of all prime numbers above that lower bound with no upper bound.

The fourth basic idea is to include the special prime number 1 as it neatly fits into the Psa. Special prime number 1 can be devided by 1 and itself (also 1). This means special prime number 1 is self reflective and thus has no multiples.

Description of the starting state (E2). The term L, L, ... ∞ is written as the periodic term $\overline{L}$ speak "period L". The periodic sieve pattern of $\mathbb{CP}_{start}$ is described by the single letter L and therefore the period length $pl_{start} = 1$ and all natural numbers $n \in \mathbb{N}$ are yet not determined.

Description of prime faculty. In this paper the products of prime numbers are used in the following way: 1, $1{\cdot}2$, $1{\cdot}2{\cdot}3$, $1{\cdot}2{\cdot}3{\cdot}5$, $1{\cdot}2{\cdot}3{\cdot}5{\cdot}7$, $1{\cdot}2{\cdot}3{\cdot}5{\cdot}7{\cdot}11$, and so on and so forth. In order to have a convenient way of writing these products, a new type of faculty is defined, called "prime faculty" and denoted by $\mathbb{P}_n$ (see example E3).

$$\mathbb{P}_n \text{ is the product of all prime numbers} \leq n \text{ with } n \in \mathbb{N} \qquad \text{XIV}$$

$$\begin{aligned}
\underset{.}{P}_1 &:= 1\\
\underset{.}{P}_2 &:= 1\cdot 2 = 2\\
\underset{.}{P}_3 &:= 1\cdot 2\cdot 3 = 6\\
\underset{.}{P}_4 &:= 1\cdot 2\cdot 3 = 6\\
\underset{.}{P}_5 &:= 1\cdot 2\cdot 3\cdot 5 = 30\\
\underset{.}{P}_7 &:= 1\cdot 2\cdot 3\cdot 5\cdot 7 = 210\\
\underset{.}{P}_{11} &:= 1\cdot 2\cdot 3\cdot 5\cdot 7\cdot 11 = 2310
\end{aligned} \tag{E3}$$

Note that $\underset{.}{P}_4 = \underset{.}{P}_3$, thus prime faculties of non prime numbers are equal to the prime faculty of the next lower prime number.

Note further that the result of $\underset{.}{P}_2$ is even.

$$\text{all prime faculties} > \underset{.}{P}_2 \text{ are even} \tag{XV}$$

Chapter 3 Periodic sieve algorithm (Psa)

The Sieve of Eratosthenes (ER) is a simple, ancient algorithm for finding all prime numbers up to a specified integer. Also, modern sieves have been derived from the Sieve of Eratosthenes. I maintain that common to all these sieves is the necessity of an upper bound — the specified integer.

In this paper an algorithm is described, that results in an infinite number of infinite periodic sieves and thus operates without an upper bound. It is called the Periodic sieve algorithm (Psa). The Psa has a well defined lower bound. Also it heavily consumes computing power even at low prime numbers. The advantage therefore is not in producing prime numbers but in its infinite nature. Thus the theoretical operation of the Psa enables new conclusions about the nature of prime numbers, its results may be useful for mathematical proofs and cryptography.

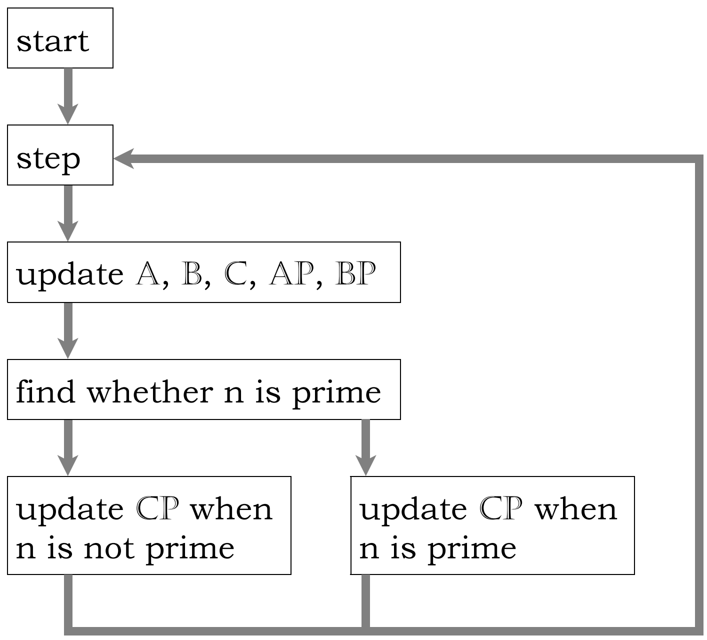


Fig. 3.1 Periodic sieve algorithm:

0.) start

There is no step number yet. The starting state as described in chapter 2 is:

$$\mathbb{A}_{start} = \varnothing \qquad \mathbb{B}_{start} = \varnothing \qquad \mathbb{C}_{start} = \{\, 1, 2, \ldots \infty \,\} \tag{XVI}$$

$$\mathbb{AP}_{start} = \varnothing \qquad \mathbb{BP}_{start} = \varnothing \qquad \mathbb{CP}_{start} = \{\, \bar{L} \,\} \tag{XVII}$$

Neither prime numbers nore multiples have been determined yet. Thus all natural numbers $n \in \mathbb{N}$ are yet undetermined and are contained in set $\mathbb{C}_{start}$.

1.) step

The element of $\mathbb{C}_{n-1}$ with the smallest value (= the leftmost element) is the new step number n. At the start $\mathbb{C}_{start}$ is $\mathbb{C}_{n-1}$. The algorithm loops onto itself and since $\mathbb{C}_{start}$ respectively $\mathbb{C}_{n-1}$ contain an infinite number of elements, the algorithm never ends and steps to infinity.

3.1) update $\mathbb{A}$

The only element of $\mathbb{B}_{n-1}$ is cut from $\mathbb{B}_{n-1}$ and pasted into $\mathbb{A}_{n-1}$. By the order described for $\mathbb{A}$ in chapter 2 the new element will always become the rightmost element. Thus $\mathbb{A}_{n-1}$ becomes $\mathbb{A}_n$. At the start $\mathbb{B}_{start} = \varnothing$ therefore $\mathbb{A}_1$ is an empty set, since there is no element in $\mathbb{B}_{start}$ to cut.

3.2) update $\mathbb{B}$

The leftmost element of $\mathbb{C}_{n-1}$ (= step number n) is cut from $\mathbb{C}_{n-1}$ and pasted into $\mathbb{B}_{n-1}$ left empty by the update of $\mathbb{A}$. Thus $\mathbb{B}_{n-1}$ again contains one element that is the step number and thus becomes $\mathbb{B}_n$. At the start $\mathbb{B}_{start} = \varnothing$ therefore as $\mathbb{B}$ is filled for the first time it becomes $\mathbb{B}_1$.

3.3) update $\mathbb{C}$

Has already taken place during the update of $\mathbb{B}$ by cutting the leftmost element of $\mathbb{C}_{n-1}$. Thus $\mathbb{C}_{n-1}$ has become $\mathbb{C}_n$.

3.4) update $\mathbb{AP}$

The only element of $\mathbb{BP}_{n-1}$ is cut from $\mathbb{BP}_{n-1}$ and pasted into $\mathbb{AP}_{n-1}$. The new element has to become the rightmost element to keep $\mathbb{AP}$ in match with $\mathbb{A}$. Thus $\mathbb{AP}_{n-1}$ becomes $\mathbb{AP}_n$. At the start $\mathbb{BP}_{start} = \varnothing$ therefore $\mathbb{AP}_1$ is an empty set, since there is no element in $\mathbb{BP}_{start}$ to cut.

3.5) update $\mathbb{BP}$

The leftmost element of $\mathbb{CP}_{n-1}$ (= type of step number n) is copied (not cut !) from $\mathbb{CP}_{n-1}$ and pasted into $\mathbb{BP}_{n-1}$ left empty by the update of $\mathbb{AP}$. Thus $\mathbb{BP}_{n-1}$ again contains one element that is the type of the step number before finding wether n is prime and thus becomes $\mathbb{BP}_n$. At the start $\mathbb{BP}_{start} = \varnothing$ therefore as $\mathbb{BP}$ is filled for the first time it becomes $\mathbb{BP}_1$.

4.) find wether n is prime

The element n, that is the step number, is contained in $\mathbb{B}_n$. Its type information is contained in $\mathbb{BP}_n$. Analogous to the ER:

If $\mathbb{BP}_n$ contains the type element M then the current step number is not prime but a multiple of a prime.

If $\mathbb{BP}_n$ contains the type element L (= undetermined) then the current step number is prime. This is denoted by changing L into P.

5.1) update CP when n is not prime

a. procedure: move (Lemma about move)

As can be seen in the description "update of $\mathbb{BP}$" the type information of the step number n is not cut from $\mathbb{CP}_{n-1}$. Instead this leftmost type information is moved to the rightmost place of the periodic term. Thus $\mathbb{CP}_{n-1}$ becomes $\mathbb{CP}_n$.

5.2) update CP when n is prime

a. procedure: move (Lemma about move)

The first procedure is the same as for "n is not prime"

$\Rightarrow$ "moved $\mathbb{CP}_{n-1}$" pattern.

b. procedure: copy

The pattern length pl_{n-1} (= period length) is increased by copying the "moved $\mathbb{CP}_{n-1}$" pattern and pasting it n-1 times to the right of itself

$\Rightarrow$ "moved and lengthened $\mathbb{CP}_{n-1}$" pattern with length pl_n.

c. procedure: change

The types of all numbers $x{\cdot}n$ (n and $x \in \mathbb{N} \;\wedge\; x > 1 \;\wedge\; n > 1$) in $\mathbb{C}_n$ are turned in the "moved and lengthened $\mathbb{CP}_{n-1}$" pattern from undertermined L types into M types, unless they are already of type M. The case n = 1 is excluded because prime number 1 is self reflective and therefore has no multiples. Thus the "moved and lengthened $\mathbb{CP}_{n-1}$" pattern becomes $\Rightarrow \mathbb{CP}_n$.

results of the Psa

The results of the first four steps are summarized here as example E4.

$\mathbb{A}_{start} := \varnothing$	$\mathbb{B}_{start} := \varnothing$	$\mathbb{C}_{start} := \{1, 2, \ldots \infty\} = \mathbb{N}$	
$\mathbb{A}_1 = \varnothing$	$\mathbb{B}_1 = \{1\}$	$\mathbb{C}_1 = \{2, 3, \ldots \infty\}$	
$\mathbb{A}_2 = \{1\}$	$\mathbb{B}_2 = \{2\}$	$\mathbb{C}_2 = \{3, 4, \ldots \infty\}$	
$\mathbb{A}_3 = \{1, 2\}$	$\mathbb{B}_3 = \{3\}$	$\mathbb{C}_3 = \{4, 5, \ldots \infty\}$	
$\mathbb{A}_4 = \{1, 2, 3\}$	$\mathbb{B}_4 = \{4\}$	$\mathbb{C}_4 = \{5, 6, \ldots \infty\}$	E4
$\mathbb{AP}_{start} := \varnothing$	$\mathbb{BP}_{start} := \varnothing$	$\mathbb{CP}_{start} := \{L, L, \ldots \infty\} = \{\overline{L}\}$	
$\mathbb{AP}_1 = \varnothing$	$\mathbb{BP}_1 = \{P\}$	$\mathbb{CP}_1 = \{\overline{L}\}$	
$\mathbb{AP}_2 = \{P\}$	$\mathbb{BP}_2 = \{P\}$	$\mathbb{CP}_2 = \{\overline{LM}\}$	
$\mathbb{AP}_3 = \{P, P\}$	$\mathbb{BP}_3 = \{P\}$	$\mathbb{CP}_3 = \{\overline{MLMLMM}\}$	
$\mathbb{AP}_4 = \{P, P, P\}$	$\mathbb{BP}_4 = \{M\}$	$\mathbb{CP}_4 = \{\overline{LMLMMM}\}$	

The results can be written similar to the ER (Fig. 3.2).

n = 2

3	4
5	6
7	8
9	10
11	12
13	14
L	**M**

etc.

p1 = 1 ×2 = 2

n = 3

4	5	6	7	8	9
10	11	12	13	14	15
16	17	18	19	20	21
22	23	24	25	26	27
28	29	30	31	32	33
34	35	36	37	38	39
M	**L**	**M**	**L**	**M**	**M**

etc.

p1 = 1 × 2 × 3 = 6

n = 4

5	6	7	8	9	10
11	12	13	14	15	16
17	18	19	20	21	22
23	24	25	26	27	28
29	30	31	32	33	34
35	36	37	38	39	40
L	**M**	**L**	**M**	**M**	**M**

etc.

p1 = 1 × 2 × 3 = 6

n = step number in $\mathbb{B}_n$

(grey box) = multiple in $\mathbb{CP}_n$ = can not be prime number

(white box) = Lücke in $\mathbb{CP}_n$ = maybe prime number

pl = period length

n = 5

6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125
126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155
156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185
M	**L**	**M**	**M**	**M**	**L**	**M**	**L**	**M**	**M**	**M**	**L**	**M**	**L**	**M**	**M**	**M**	**L**	**M**	**M**	**M**	**M**	**M**	**L**	**M**	**L**	**M**	**M**	**M**	**M**

etc.

p1 = 1 × 2 × 3 × 5 = 30

Fig. 3.2 Some infinite periodic sieves written similar to the Sieve of Eratosthenes (ER).

Chapter 4 Explanations to the Psa

In the following illustrations are given for some of the steps in the Psa.

The starting state can be illustrated as in Fig. 4.1 by a representation similar to the ER.

In Fig. 4.2 the infinite periodic sieve n = 1 is shown. It differs from Fig. 4.1 in that it starts with 2 instead of 1.

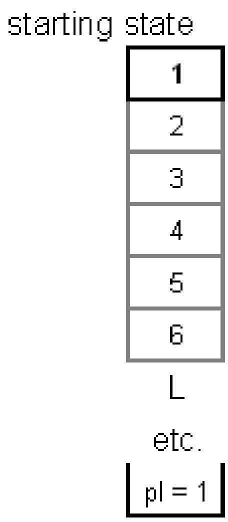


Fig. 4.1 start

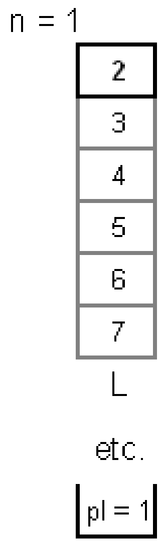


Fig. 4.2 n = 1

Writing step n = 1 as a number and pattern ray in Fig. 4.3

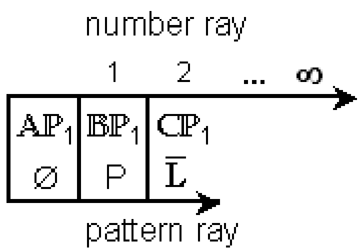


Fig. 4.3 n = 1 in ray view

With regards to chapter 3 steps 5.1.a and 5.2.a, the “procedure: move” is equivalent to cutting the first element of the set $\mathbb{C}$ and therefore set $\mathbb{CP}$ remains matched with set $\mathbb{C}$. This equivalence is shown in “Lemma about move” (E5).

Periodic pattern of n = 3:	**M**LMLMM
This means:	**M**LMLMM, **M**LMLMM, **M**LMLMM, **M**… ∞
Cutting the first letter leaves:	LMLMM, **M**LMLMM, **M**LMLMM, **M**… ∞
This can be written as:	LMLMM**M**, LMLMM**M**, LMLMM**M**, … ∞
Thus moved (n = 4):	LMLMM**M**

E5
(Lemma about move)

With regards to chapter 3 step 5.2.b, each prime number p > 1 determines that all natural numbers x·p with $x \in \mathbb{N}$, $p \in \mathbb{N}$ and x > 1 are not prime (multiples).

XVIII

Thus (XVIII) each prime number p creates a periodic pattern of M type numbers in $\mathbb{CP}$ with period length $pl_{additional}$ (XIX). The "moved $\mathbb{CP}_{n-1}$" pattern (chapter 3 step 5.2.a) has to be combined with the new pattern generated by p. When two patterns are combined the period length of the combined pattern is the least common multiple of the two period lengths (wave functions).

$$pl_{combined} = pl_{previous} \cdot pl_{additional} \tag{XIX}$$

$$pl_n = pl_{n-1} \cdot pl_{only\ n} = \underset{\cdot}{P}_n \quad \text{with} \qquad n \in \mathbb{N} \tag{XX}$$

Thus the pattern length respectively period length is increased by copying the "moved $\mathbb{CP}_{n-1}$" pattern and pasting it n-1 times to the right of itself. This means the "moved and lengthened $\mathbb{CP}_{n-1}$" pattern is n times as long as before.

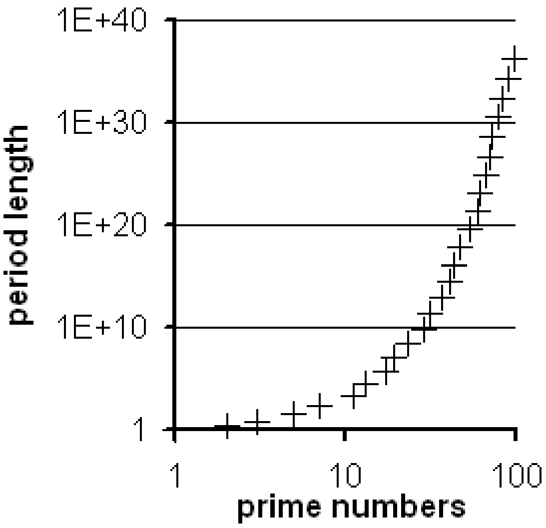


Fig. 4.4 prime numbers < 100

Fig. 4.4 has a double log scale. Periodic patterns in $\mathbb{CP}$ become very large even at low prime numbers. Pattern/period lengths in E6.

$$\begin{aligned}
pl_3 &= \underset{\cdot}{P}_3 &&= 6 \\
pl_{11} &= \underset{\cdot}{P}_{11} &&= 2310 \\
pl_{13} &= \underset{\cdot}{P}_{13} &&= 30030 \\
pl_{31} &= \underset{\cdot}{P}_{31} &&= 2{,}0056\text{E}+11 \\
pl_{739} &= \underset{\cdot}{P}_{739} &&= 3{,}9082\text{E}+306
\end{aligned} \tag{E6}$$

Chapter 5 Prime number functions — Latisses

In each infinite periodic sieve there are one or more L columns, see Fig. 5.1 and also Fig. 4.1, 4.2, and 3.2. The L columns are — by definition of L types — the only columns that hold prime numbers, thus L columns are columns of possible prime numbers (= Hüllkurve in Physics). This also means not all numbers in L columns are prime numbers.
For all step numbers n > 1 the infinite periodic sieves have an even number of columns (XV, XXI).

$$pl_n = P_n \quad \text{with } n \in \mathbb{N} \text{ and } n > 1 \qquad \text{XXI}$$

Because of the even number of columns for step numbers n > 1 the infinite periodic sieves n > 1 have the following property. Each column contains either even or uneven numbers. And from the Psa follows all numbers in M columns are not prime Fig. 5.1.

H_4 L columns M column with uneven numbers

4	5	6	7	8	9
10	11	12	13	14	15
16	17	18	19	20	21
22	23	24	25	26	27
28	29	30	31	32	33
34	35	36	37	38	39
M	**L**	**M**	**L**	**M**	**M**

head numbers h_{column}

Fig. 5.1 Part of number ray (Fig. 4.3)
Infinite periodic sieve $\mathbb{CP}_3$
with n = 3 and $pl_n = 6$

The numbers in each column can be described by prime number functions. The functions follow by logic from the infinite periodic sieves.

$$f_{(x,\, pl_n,\, h_{column})} := pl_n \cdot x + h_{column} \qquad \text{with } n \in \mathbb{N} \text{ and } x \in \mathbb{N}_0 \qquad \text{XXII}$$

The special head number $^{H}h_{column}$ is the head number in the left most column, pl_n is the period length = pattern length = number of columns of the infinite periodic sieve n.

At x = 0 the function is equal to $f_{(x,\, pl_n,\, h_{column})} = h_{column}$ XXIII

In the following, examples are given for the prime number function in E7 and Fig. 3.2.

With $n \in \mathbb{N}$ and $x \in \mathbb{N}_0$:

$$f_{(x, pl_n, h_{column})} := pl_n \cdot x + h_{column}$$

pl_2 set $n = 2$

$$f_{(x,2,3)} = 2 \cdot x + 3$$

pl_6 sets $n = 3$ and $n = 4$

$$f_{(x,6,5)} = 6 \cdot x + 5$$
$$f_{(x,6,7)} = 6 \cdot x + 7$$

pl_{30} sets $n = 5$ and $n = 6$

$$f_{(x,30,7)} = 30 \cdot x + 7$$
$$f_{(x,30,11)} = 30 \cdot x + 11$$
$$f_{(x,30,13)} = 30 \cdot x + 13$$
$$f_{(x,30,17)} = 30 \cdot x + 17$$
$$f_{(x,30,19)} = 30 \cdot x + 19$$
$$f_{(x,30,23)} = 30 \cdot x + 23$$
$$f_{(x,30,29)} = 30 \cdot x + 29$$
$$f_{(x,30,31)} = 30 \cdot x + 31$$

E7

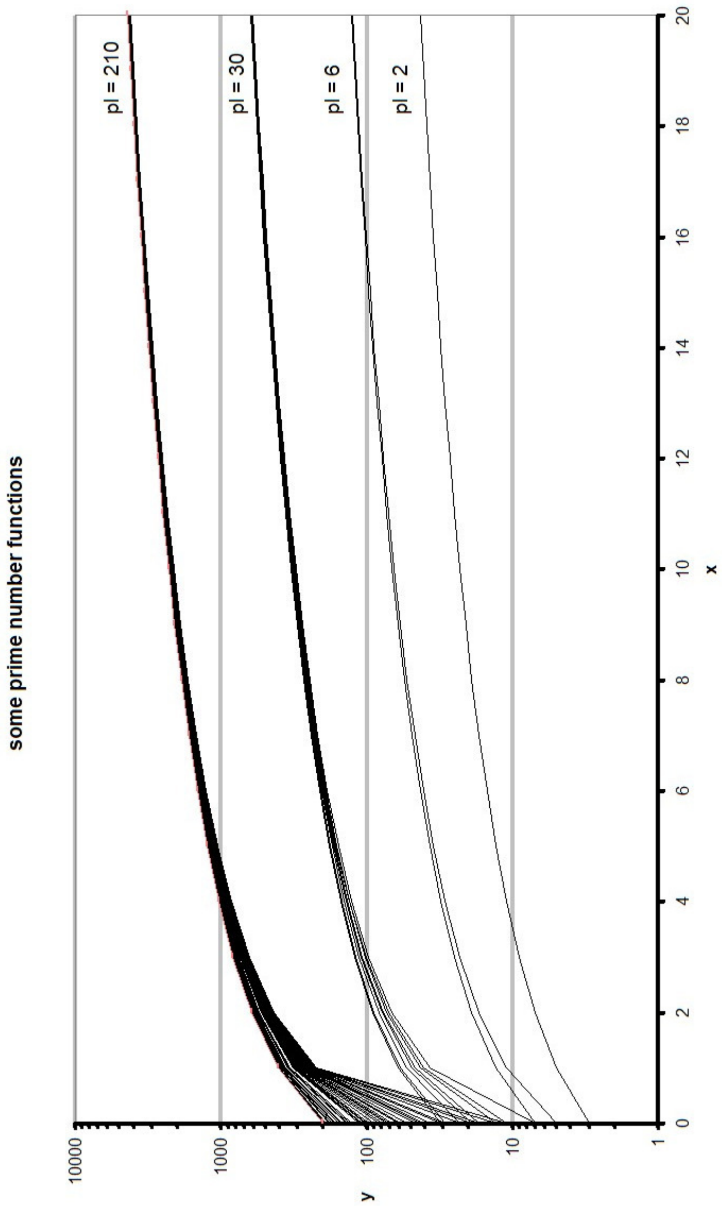
Hüllkurve

The prime number function describes the L columns (Fig. 5.1) of the infinite periodic sieves and results not only in prime numbers but also in non-prime numbers (together possible prime numbers). This (each set) is the Hüllkurve (like in Physics).

Prime number function theorem

Each single infinite periodic sieve $\mathbb{CP}_n$ of Psa contains all prime numbers > n to infinity, with $n \in \mathbb{N}$ and these prime numbers are located in the L columns only. Therefore by logic all prime numbers > n to infinity have to fulfill exactely one (Fig. 5.2) specific prime number function of each set of prime number functions belonging to periodic sieves $\mathbb{CP}_n$ (E7). Since there are an infinite number of infinite periodic sieves created by the Psa, which therefore all have the same properties, this holds true for all periodic sieves $\mathbb{CP}_n$, with $n \in \mathbb{N}$.

XXIV



In Fig. 5.2 the y-axis has a log scale.

Fig. 5.2
some prime number functions
set $pl_n = 2$
sets $pl_n = 6$
sets $pl_n = 30$
sets $pl_n = 210$

Latisses (speak: Lateeses)

Given the examples in E7 the following is hypothesized (for example):

$$f_{(x,6,7)} = f_{(x,6,5)} \cdot f_{(x,6,5)} \quad \text{XXV}$$

$$f_{(x,6,7)} = f_{(x,6,7)} \cdot f_{(x,6,7)} \quad \text{XXVI}$$

$$f_{(x,6,5)} = f_{(x,6,5)} \cdot f_{(x,6,7)} = f_{(x,6,7)} \cdot f_{(x,6,5)} \quad \text{XXVII}$$

It is further hypothesized that Latisses are copied (multiplied) and changed with each Psa sieve. XXVIII

Chapter 6 Six pack structures – four pattern letters

At each step number n with $n \in \mathbb{N}$ a periodic pattern consisting of L and M types can be described for sets $\mathbb{CP}_n$, resulting in the periodic sieves. Since each periodic sieve $\mathbb{CP}_n$ is infinite its pattern must be contained in some form in all subsequent periodic sieves $\mathbb{CP}_{x>n}$ with $x \in \mathbb{N}$ and $n \in \mathbb{N}$.

As already discussed earlier the period length pl_n and thus pattern length will increase very rapidly with n (chapter 4) and therefore the patterns become difficult to handle in practical terms. In turn the short patterns at very low step numbers n are easy to display graphically and thus to draw conclusions from. Here the pattern Fig. 6.1 at step number n = 4 is picked to describe a feature of all infinite Periodic Sieves $\mathbb{CP}_n$ with $n \geq 4$ and $n \in \mathbb{N}$.

n = 4

5	6	7	8	9	10
11	12	13	14	15	16
17	18	19	20	21	22
23	24	25	26	27	28
29	30	31	32	33	34
35	36	37	38	39	40
L	**M**	**L**	**M**	**M**	**M**

etc.

Fig. 6.1 infinite periodic sieve
n = 4 and $pl_n = 6$

Definition of letter a:

a := LMLMMM speak: "letter a" XXIII

At subsequent step numbers $n \geq 4$ with $n \in \mathbb{N}$, copies of letter a ("procedure: copy" in chapter 3) can be possibly changed ("procedure: change" in chapter 3) in only three ways, which follows directly by logic:

1. The second L is turned into M.
2. The first L is turned into M.
3. Both L's are turned into M.

The letters b, c, and d are thus defined:

b := LMMMMM speak: "letter b" XXIV

c := MMLMMM speak: "letter c" XXV

d := MMMMMM speak: "letter d" XXVI

XXVII

Therefore all patterns of sets $\mathbb{CP}_n$ with $n \geq 4$ and $n \in \mathbb{N}$ consist of only four letters a, b, c, and d each being a six pack of L and M types.

For practical reasons an offset < 6 preciding the beginning of the pattern period has to be allowed for. The offset avoids redefinition of letters a, b, c, and d.

Only letter a contains a twin Lücke L and thus only letter a can produce a prime twin. Of couse not all twin Lücken will produce twin prime numbers, they just hold the potential to do so.

The numbers 5 and 7 get their triplet partner 3 because prime number 3 is created before the "four letter six pack structure" has build when stepping through $\mathbb{N}$ by the Psa.

Proof about prime triplets:
It holds true that no matter in which sequence the letters a, b, c, and d are arranged it is impossible to build a triple Lücke L and therefore it is impossible that there are any prime number triplets beyond the first and only prime triplet 3, 5, and 7.

q. e. d.

At step numbers $n > 4$ with $n \in \mathbb{N}$ therefore: period length $pl_{\text{only } n} > 4$. Thus it is impossible that both L's in any letter a will be turned into M at the same step number $n > 4$. This means letter a can turn into b and c but not d. Having only one L at different locations letters b and c can each turn into d but not into each other. Letter d does not contain any L (prime gap) and therefore can not turn into any other letter, see Fig. 6.2.

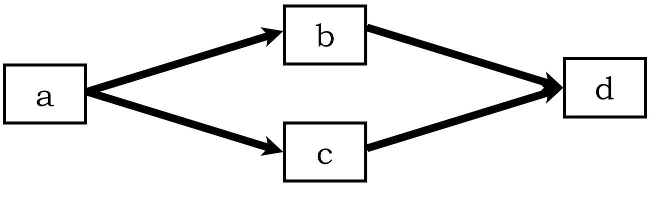


Fig. 6.2 which letter can turn into which

Chapter 7 Prime gaps

The letter d and d-aggregates contain only multiples and thus are prime gaps. From the Psa it holds true that there are an infinite number of prime gaps.

Proof about prime gaps:

As the Periodic Sieve algorithm steps through $\mathbb{N}$ to infinity all prime gaps are transferred by Psa procedures move and copy from $\mathbb{CP}_n$ to the next set $\mathbb{CP}_{n+1}$ thus any prime gap can never be undone by stepping to infinity with $n \in \mathbb{N}$.

q. e. d.

From the proof about prime gaps we can learn that all prime gaps of any size that have been found until now or will be found in future times, are located on the pattern ray (Fig. 4.3) at the appropriate $n \in \mathbb{N}$ and will reoccure with at least their minimum size on the pattern ray and thus also on the number ray (Fig. 4.3) as the Psa steps to infinity.

Chapter 8 Expanding fractal

Fractals can be generated from a starting state by an iterative process which contains constant “reduction, copy, paste procedures" for each iterative step (Mehrfach Verkleinerungs Kopier Maschine, Peitgen and Jürgens, 1988).

The Psa and thus the nature of prime numbers is special in that there is no known reduction step.

The Psa and thus the nature of prime numbers is also special in the way that the "copy procedure" is not constant but is increased with each prime number found.

This leads to the understanding that the Psa is an expanding fractal (Mehrfach Vergrößerungs Kopier Maschine). I call the Psa also Prime Number Maschine (Primzahlmaschine).

Chapter 9 Infinite sets

According to E4 and the infinite Erastothenes type sieves in Fig. 4.1, Fig. 4.2, and Fig. 3.2 the Psa allows to calculate with infinite sets with $n \in \mathbb{N}$.

$$\{\infty\}_{\mathbb{N}} = \mathbb{A}_1 + \mathbb{B}_1 + \{\infty\}_{\mathbb{C}_1} = \mathbb{A}_2 + \mathbb{B}_2 + \{\infty\}_{\mathbb{C}_2} = \mathbb{A}_n + \mathbb{B}_n + \{\infty\}_{\mathbb{C}_n} \qquad \text{XXVIII}$$

$$\begin{aligned} \{\infty\}_{\mathbb{N}} &= \{\infty\}_{\mathbb{H}_1} && \text{XXIX} \\ &= \{\infty\}_{\mathbb{H}_2} + 1 \\ &= 2\ \{\infty\}_{\mathbb{H}_3} + 2 \\ &= 2\cdot 3\ \{\infty\}_{\mathbb{H}_4} + 3 \\ &= 2\cdot 3\ \{\infty\}_{\mathbb{H}_5} + 4 \\ &= 2\cdot 3\cdot 5\ \{\infty\}_{\mathbb{H}_6} + 5 \\ &= \underset{\cdot}{\mathbb{P}}_n\ \{\infty\}_{\mathbb{H}_{n+1}} + n && \text{XXX} \end{aligned}$$

Chapter 10 Conclusion

ideas

The lemma about move, Latisses, and the calculation with infinite sets are new mathematics.

outlook

The Psa touches many areas of mathematics and probably is a crossroads to link different areas of mathematics.

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